

# Inflation With Realistic Supersymmetric $SO(10)$

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## Abstract

We implement inflation within a realistic supersymmetric  $SO(10)$  model in which the doublet-triplet splitting is realized through the Dimopoulos-Wilczek mechanism, the MSSM  $\mu$  problem is resolved, and higgsino mediated dimension five nucleon decay is heavily suppressed. The cosmologically unwanted topological defects are inflated away, and from  $\delta T/T$ , the  $B - L$  breaking scale is estimated to be of order  $10^{16} - 10^{17}$  GeV. Including supergravity corrections, the scalar spectral index  $n_s = 0.99 \pm 0.01$ , with  $|dn_s/d\ln k| \lesssim 10^{-3}$ .

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In an attractive class of supersymmetric (SUSY) models inflation is associated with spontaneous breaking of a gauge symmetry, such that  $\delta T/T$  is proportional to  $(M/M_{\text{Planck}})^2$ , where  $M$  denotes the symmetry breaking scale and  $M_{\text{Planck}} (\equiv 1.2 \times 10^{19} \text{ GeV})$  denotes the Planck mass [1, 2]. Thus, from measurements of  $\delta T/T$ ,  $M$  is estimated to be of order  $10^{16} \text{ GeV}$  [1, 3, 4]. The scalar spectral index  $n_s$  in these models is very close to unity<sup>3</sup> in excellent agreement with recent fits to the data [5].<sup>4</sup> A  $U(1)$   $R$ -symmetry plays an essential role in the construction of these inflationary models. These models possess another important property, namely with the minimal Kähler potential, the supergravity (SUGRA) corrections do not spoil the inflationary scenario [2, 8], which has been realized with a variety of attractive gauge groups including  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} (\equiv G_{LR})$  [9],  $SU(4)_c \times SU(2)_L \times SU(2)_R (\equiv G_{422})$  [10] and  $SU(5)$  [11]. [The gauge symmetries  $G_{LR}$  and  $G_{422}$  were first introduced in Refs. [12, 13].] Our goal in this paper is to implement inflation in a realistic  $SO(10)$  model.

$SO(10)$  [14] has two attractive features that it shares with  $G_{422}$ , namely, it predicts the existence of right handed neutrinos as well as the seesaw mechanism. These two features are very helpful in understanding neutrino oscillations [15] and also in generating a baryon asymmetry via leptogenesis [16]. Furthermore, at least within a four dimensional setting, it seems easier to realize doublet-triplet (DT) splitting without fine tuning in  $SO(10)$  (say via the Dimopoulos-Wilczek mechanism [17]) than in  $SU(5)$ .

To implement  $SO(10)$  inflation we would like to work with a realistic model with the following properties: DT splitting is realized without fine tuning, and the low energy theory coincides with the minimal supersymmetric standard model (MSSM).

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<sup>3</sup>Following Ref. [4], that including supergravity corrections, these model can yield a spectral index somewhat larger than unity. It is in our current model that  $n_s = 0.99 \pm 0.01$ , as we emphasize in the abstract.

<sup>4</sup>Earlier there was some (weak) evidence for a running spectral index with  $dn_s/d\ln k \approx -5 \times 10^{-2}$ . But this is not confirmed by a more recent analysis [6]. We will consider the simplest models which yield  $dn_s/d\ln k \lesssim -10^{-3}$ . However, more complicated scenarios with two or more inflationary epoch can yield a significantly larger  $dn_s/d\ln k$  [7].

[For  $SO(10)$  inflation in a five dimensional setting, see Ref. [18].] The MSSM  $\mu$  problem should also be resolved, and higgsino mediated dimension five nucleon decay should be adequately suppressed. Gauge boson mediated nucleon decay is still present with a predicted nucleon lifetime of order  $10^{34} - 10^{36}$  yrs. Finally, matter parity is unbroken, so that the LSP is stable and makes up the dark matter in the universe.

To achieve natural DT splitting and the MSSM at low energies with  $SO(10)$ , one is led to consider a non-minimal set of Higgs superfields. This is to be contrasted with the subgroups of  $SO(10)$ , such as  $G_{LR}$  or  $G_{422}$  above, in which the DT splitting problem is absent. Many authors have previously addressed the DT splitting and the dimension five nucleon decay problem in  $SO(10)$  [19, 20, 21], and the proposed solutions are not necessarily straightforward. In this paper we will follow Refs. [20, 21], with suitable modifications needed to make the scheme consistent with the desired inflationary scenario, and also to avoid potential cosmological problems (monopoles, moduli, etc). While doing this we would like to also ensure that the SUGRA corrections also do not disrupt the inflationary scenario.

A minimal set of Higgs required to break  $SO(10)$  to the MSSM gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y (\equiv G_{SM})$  is  $\mathbf{45}_H$ ,  $\mathbf{16}_H$ ,  $\overline{\mathbf{16}}_H$ . A non-zero vacuum expectation value (VEV) of  $\mathbf{45}_H$  along the  $B-L$  ( $I_{3R}$ ) direction breaks  $SO(10)$  to  $G_{LR}$  ( $SU(4)_c \times SU(2)_L \times U(1)_R$ ) and produces magnetic monopoles. The  $\mathbf{16}_H$ ,  $\overline{\mathbf{16}}_H$  VEVs break  $SO(10)$  to  $SU(5)$  and induce masses for the right handed neutrinos via dimension five operators. [Note that breaking of  $SO(10)$  with  $\mathbf{16}_H + \overline{\mathbf{16}}_H$ , in contrast to  $\mathbf{126}_H + \overline{\mathbf{126}}_H$ , does not produce  $Z_2$  cosmic strings [22].] One of our goals, of course, is to make sure that the topological defects do not pose cosmological difficulties. Thus, it would be helpful if during inflation  $SO(10)$  is, for instance, broken to  $G_{LR}$  [23],  $SU(4)_c \times SU(2)_L \times U(1)_R$ , or  $G_{SM}$ .

To implement DT splitting without fine tuning and eliminate dimension five proton decay, and to recover the MSSM at low energies with the  $\mu$  problem resolved, we need an additional  $\mathbf{45}$ -plet ( $\mathbf{45}'_H$ ), two additional  $\mathbf{16} + \overline{\mathbf{16}}$  pairs, two  $\mathbf{10}$ -plets ( $\mathbf{10}_h$

and **10**), and several singlets [20, 21]. One more **45**-plet is also required by  $U(1)$   $R$ -symmetry. This symmetry, among other things, plays an essential role in realizing inflation, and its  $Z_2$  subgroup coincides with the MSSM matter parity. The  $SO(10)$  singlet superfields are denoted as  $S$ ,  $X$ ,  $X'$ ,  $Y$ ,  $P$ ,  $\overline{P}$ ,  $Q$ , and  $\overline{Q}$ , whose roles will be described below. Table I displays the quantum numbers (under the global  $U(1)$   $R$  and  $U(1)_A$  symmetries) of all the Higgs sector superfields and the third family matter field (**16**<sub>3</sub>). Following standard practice, we employ the same notation for the superfields and their scalar components.

	$S$	$X$	$X'$	$Y$	$P$	$\overline{P}$	$Q$	$\overline{Q}$	<b>10</b>	<b>10</b> <sub><math>h</math></sub>
$R$	1	-1	-1	0	0	0	-2	2	1	0
$A$	0	-2/3	-2/3	-1/3	-1/4	1/4	-1/2	1/2	1/6	0
	<b>16</b>	$\overline{\mathbf{16}}$	<b>16'</b>	$\overline{\mathbf{16'}}$	<b>16</b> <sub><math>H</math></sub>	$\overline{\mathbf{16}}$ <sub><math>H</math></sub>	<b>16</b> <sub>3</sub>	<b>45</b>	<b>45</b> <sub><math>H</math></sub>	<b>45'</b> <sub><math>H</math></sub>
$R$	1	3	2	2	0	0	1/2	1	0	-1
$A$	1/2	2/3	2/3	2/3	0	0	0	1/2	-1/6	-1/3

Table I

To break  $SO(10)$  to  $G_{LR}$ , consider the superpotential,

$$\begin{aligned}
W_{45} = & \frac{\alpha}{6M_*} X^{(\prime)} Y \text{Tr}(\mathbf{45}\mathbf{45}) - \frac{\beta}{6} Y \text{Tr}(\mathbf{45}\mathbf{45}_H) \\
& + \frac{\gamma_1}{36M_*} \text{Tr}(\mathbf{45}\mathbf{45}_H) \text{Tr}(\mathbf{45}_H\mathbf{45}_H) + \frac{\gamma_2}{6M_*} \text{Tr}(\mathbf{45}\mathbf{45}_H\mathbf{45}_H\mathbf{45}_H) ,
\end{aligned} \tag{1}$$

where  $\alpha$ ,  $\beta$ ,  $\gamma_{1,2}$  are dimensionless parameters, and  $M_*$  ( $\sim 10^{18}$  GeV) denotes the cutoff scale. As will be explained,  $X$ ,  $X'$ , and  $Y$  can develop non-zero VEVs,  $\langle X \rangle \sim \langle X' \rangle \sim \langle Y \rangle \sim 10^{16}$  GeV. Due to non-zero  $\langle Y \rangle$ , **45** <sub>$H$</sub>  can also obtain a VEV in the  $B - L$  direction from the  $\beta$  and  $\gamma_{1,2}$  terms of Eq. (1),

$$\langle \mathbf{45}_H \rangle = \left( \begin{array}{c|c} v & \\ \hline v & \\ \hline v & 0 \\ \hline & 0 \end{array} \right) \otimes i\sigma_2 , \quad \text{and} \quad \langle \mathbf{45} \rangle = 0 , \tag{2}$$

where  $v = \sqrt{\frac{\beta}{\gamma} \langle Y \rangle M_*} \equiv M_{GUT}$  ( $\approx 3 \times 10^{16}$  GeV), with  $\gamma \equiv \gamma_1 + \gamma_2$ . The  $3 \times 3$  block corresponds to  $SU(3)_c$  and the  $2 \times 2$  block to  $SU(2)_L$  of the MSSM gauge group.

Hence, the  $SO(10)$  gauge symmetry is broken to  $G_{LR}$ . Note that from the ‘ $\alpha$  term,’ the **45** multiplet becomes superheavy. It acquires a VEV of order  $(m_{3/2}M_{GUT})/M_*$  after SUSY breaking, where  $m_{3/2}$  ( $\sim$  TeV) denotes the scale of the soft parameters.

The next step in the breaking to the MSSM gauge group  $G_{SM} (= G_{LR} \cap SU(5))$  is achieved with the following superpotential,

$$\begin{aligned} W_{16} = & S \left[ \kappa \mathbf{16}_H \overline{\mathbf{16}}_H + \lambda \mathbf{10}_h \mathbf{10}_h - \kappa M_{B-L}^2 \right] - \frac{\rho}{M_*^2} S (\mathbf{16}_H \overline{\mathbf{16}}_H)^2 \\ & + \mathbf{16} \left[ \frac{\lambda_1}{M_*} \mathbf{45}_H Y - \frac{\lambda_2}{M_*} P^2 \right] \overline{\mathbf{16}}_H + \overline{\mathbf{16}} \left[ \frac{\lambda_3}{M_*} \mathbf{45}_H Q - \frac{\lambda_4}{M_*} (\mathbf{45}'_H)^2 \right] \mathbf{16}_H \\ & + \mathbf{16}' \left[ \frac{\lambda_5}{M_*} \mathbf{45}'_H Y - \lambda_6 X \right] \overline{\mathbf{16}}_H + \overline{\mathbf{16}}' \left[ \frac{\lambda_7}{M_*} \mathbf{45}'_H Y - \lambda_8 X' \right] \mathbf{16}_H, \end{aligned} \quad (3)$$

where  $\rho$  is a dimensionless coupling constant. The dimensionful parameter  $M_{B-L}$ , as determined from inflation  $(\delta T/T)$ , turns out to be of order  $10^{16} - 10^{17}$  GeV [4]. The superfield  $\mathbf{10}_h$  includes the two MSSM Higgs doublets. As previously mentioned, additional **16**,  $\overline{\mathbf{16}}$  are essential to stabilize the VEV of  $\mathbf{45}_H$  in Eq. (2) [20]. From the  $\kappa$  and  $\rho$  terms,  $\mathbf{16}_H$  and  $\overline{\mathbf{16}}_H$  develop VEVs of order  $M_{B-L}$ , breaking  $SO(10)$  to  $SU(5)$ ,

$$|\langle \mathbf{16}_H \rangle|^2 = |\langle \overline{\mathbf{16}}_H \rangle|^2 = \frac{M_{B-L}^2}{2\zeta} \left[ 1 - \sqrt{1 - 4\zeta} \right], \quad (4)$$

where  $\zeta \equiv \rho M_{B-L}^2 / (\kappa M_*^2)$  [10], while  $\langle S \rangle = \langle \mathbf{10}_h \rangle = 0$  upto corrections of  $O(m_{3/2})$  by including soft SUSY breaking terms in the scalar potential [9]. The ‘‘D-term’’ scalar potential vanishes along the (D-flat) direction  $|\langle \mathbf{16}_H \rangle| = |\langle \overline{\mathbf{16}}_H^* \rangle| (= |\langle \overline{\mathbf{16}}_H \rangle|)$ . Together with Eq. (2), the  $SO(10)$  gauge symmetry is broken to the MSSM gauge symmetry. The MSSM Higgs doublets arise from  $\mathbf{10}_h$ . With  $\langle S \rangle \approx -m_{3/2}/\kappa$ , the  $\mu$  term from Eq. (3) is of order  $(\lambda/\kappa)m_{3/2} \sim$  TeV, for  $\kappa \approx \lambda$ .<sup>5</sup> Similarly the soft term  $B\mu$  ( $\approx -2(\lambda/\kappa)m_{3/2}^2$ ) is generated [9, 25].

Our next step is to ensure that the low energy theory coincides precisely with the MSSM. With  $SO(10)$  broken to  $G_{LR}$  by  $\langle \mathbf{45}_H \rangle$  via Eq. (1), the goldstone modes

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<sup>5</sup>From the non-renormalizable term  $y_\mu \mathbf{1010}_h \langle \mathbf{16}_H \mathbf{45}_H \overline{\mathbf{16}}_H \rangle / M_*^2$ , the doublets in  $\mathbf{10}_h$  obtains a ‘‘seesaw mass’’  $y_\mu^2 \langle (\mathbf{16}_H \mathbf{45}_H \overline{\mathbf{16}}_H) \rangle^2 / (M_*^4 \langle \mathbf{45}'_H \rangle) \sim$  TeV with  $y_\mu \sim 10^{-3}$ , which modifies the  $\mu$  parameter at low energies.

from  $\mathbf{45}_H$ ,  $[\{(\mathbf{3}, \bar{\mathbf{2}})_{-5/6}, (\mathbf{3}, \mathbf{2})_{1/6}, (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}\} + \text{h.c.}]$  in terms of  $G_{SM}$ , are absorbed by the gauge sector. The states of  $(\mathbf{8}, \mathbf{1})_0$ ,  $(\mathbf{1}, \mathbf{3})_0$ ,  $(\mathbf{1}, \mathbf{1})_0$ ,  $(\mathbf{1}, \mathbf{1})_1$ , and  $(\mathbf{1}, \mathbf{1})_{-1}$  contained in  $\mathbf{45}_H$  acquire superheavy masses through the quartic couplings. On the other hand, when  $SO(10)$  breaks to  $SU(5)$  by  $\langle \mathbf{16}_H \rangle$  and  $\langle \bar{\mathbf{16}}_H \rangle$ , the states  $[\{(\mathbf{3}, \mathbf{2})_{1/6}, (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}, (\mathbf{1}, \mathbf{1})_1\} + \text{h.c.}] + (\mathbf{1}, \mathbf{1})_0$  in  $\mathbf{16}_H, \bar{\mathbf{16}}_H$  should be absorbed by the gauge sector, while  $[\{(\bar{\mathbf{3}}, \mathbf{1})_{1/3}, (\mathbf{1}, \bar{\mathbf{2}})_{-1/2}\} + \text{h.c.}]$  remain massless (or light). Note that  $[\{(\mathbf{3}, \mathbf{2})_{1/6}, (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}\} + \text{h.c.}]$  are common between  $\mathbf{45}_H$  and  $\mathbf{16}_H, \bar{\mathbf{16}}_H$ . Thus, when  $SO(10)$  breaks to  $G_{SM}$  by an adjoint and a vector-like pair of spinorial Higgs, the superfields associated with  $[\{(\mathbf{3}, \mathbf{2})_{1/6}, (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}, (\bar{\mathbf{3}}, \mathbf{1})_{1/3}, (\mathbf{1}, \bar{\mathbf{2}})_{-1/2}\} + \text{h.c.}]$  are pseudo-goldstone modes. The extra light multiplets would spoil the unification of the MSSM gauge couplings, and therefore must be eliminated.

The simplest way to remove them from the low energy spectrum is to introduce couplings such as  $\mathbf{16}_H \mathbf{45}_H \bar{\mathbf{16}}_H$ . However, the presence of such a term in the superpotential destabilizes the form of  $\langle \mathbf{45}_H \rangle$  given in Eq. (2), in such a way that at the SUSY minimum,  $v = 0$  is required. It was shown in Ref. [20] that with the ‘ $\lambda_i$ ’ couplings ( $i = 1, 2, 3, 4$ ) and an additional  $\mathbf{16} - \bar{\mathbf{16}}$  pair in Eq. (3), the unwanted pseudo-goldstone modes all become superheavy, keeping intact the form of Eq. (2) at the SUSY minimum.

From the ‘F-flat conditions’ with  $\mathbf{16}_H$  and  $\bar{\mathbf{16}}_H$  acquiring non-zero VEVs, one finds

$$\langle \mathbf{45}_H \rangle \langle Y \rangle = \frac{\lambda_2}{\lambda_1} \langle P^2 \rangle, \quad \langle \mathbf{45}_H \rangle \langle Q \rangle = \frac{\lambda_4}{\lambda_3} \text{Tr} \langle \mathbf{45}'_H \rangle^2. \quad (5)$$

Thus, if  $P$  and  $Q$  develop VEVs,  $\langle \mathbf{45}_H \rangle$ ,  $\langle \mathbf{45}'_H \rangle$ , and  $\langle Y \rangle$  should also appear. We will soon explain how  $\langle P \rangle$  and  $\langle Q \rangle$  arise. Since  $\langle Y \rangle$  is related to  $\langle \mathbf{45}_H \rangle$  via Eq. (2), both are uniquely determined. We assume that  $\langle \mathbf{45}'_H \rangle$  points in the  $I_{3R}$  direction,

$$\langle \mathbf{45}'_H \rangle = \left( \begin{array}{c|c} 0 & \\ \hline 0 & \\ \hline 0 & v' \\ & v' \end{array} \right) \otimes i\sigma_2. \quad (6)$$

Recall that  $\langle \mathbf{45}'_H \rangle$  is employed to suppress higgsino mediated dimension five nucleon decay [21]. Similarly, due to the presence of the ‘ $\lambda_i$ ’ ( $i = 5, 6, 7, 8$ ) couplings in Eq. (3), the low energy spectrum is protected even with the  $\mathbf{45}'_H$  present [21]. With non-zero VEVs for  $\mathbf{45}'_H$  and  $Y$ ,  $X$  and  $X'$  slide to values satisfying

$$\frac{\lambda_{5,7}}{M_*} \langle \mathbf{45}'_H \rangle \langle Y \rangle - \lambda_{6,8} \langle X^{(\prime)} \rangle = 0 , \quad (7)$$

with  $|\langle \mathbf{16}' \rangle| = |\langle \mathbf{16} \rangle| \sim O(m_{3/2})$ . In order to guarantee the ‘ $\lambda_i$ ’ couplings in Eq. (3) and to forbid  $\mathbf{16}_H \mathbf{45}_H \mathbf{16}_H$ , the  $U(1)$  symmetries in Table I are essential.

To obtain non-vanishing VEVs for  $P$  and  $Q$ , one could, as a simple example, consider the following superpotential,

$$W_{PQ} = S \left[ \kappa_1 P \overline{P} + \kappa_2 Q \overline{Q} \right] - \frac{S}{M_*^2} \left[ \rho_1 (P \overline{P})^2 + \rho_2 (Q \overline{Q})^2 \right] , \quad (8)$$

such that

$$\langle P \overline{P} \rangle = \frac{\kappa_1}{\rho_1} M_*^2 \sim M_{GUT}^2 , \quad \langle Q \overline{Q} \rangle = \frac{\kappa_2}{\rho_2} M_*^2 \sim M_{GUT}^2 . \quad (9)$$

The  $\lambda_{2,3}$  terms in Eq. (3) just determine  $\langle \mathbf{45}_H \rangle$ ,  $\langle Y \rangle$ , and  $\langle \mathbf{45}'_H \rangle$ . With the inclusion of soft SUSY breaking terms, the VEVs  $\langle P \rangle$ ,  $\langle \overline{P} \rangle$ ,  $\langle Q \rangle$ , and  $\langle \overline{Q} \rangle$  would be completely fixed. To avoid potential cosmological problems associated with moduli fields, we make the important assumption that the VEVs satisfy the constraints  $\langle P \rangle = \langle \overline{P} \rangle$  and  $\langle Q \rangle = \langle \overline{Q} \rangle$ . This could be made plausible by assuming universal soft scalar masses, and that the SUSY breaking “A-terms” asymmetric under  $P \leftrightarrow \overline{P}$  and  $Q \leftrightarrow \overline{Q}$  are plausibly small enough.<sup>6,7</sup> Note that even with the soft SUSY breaking terms in the Lagrangian, the GUT scale results Eqs. (5) and (9) should be still effectively valid.

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<sup>6</sup>In gravity mediated SUSY breaking scenario with the minimal Kähler potential, “A-terms” are given by  $m_{3/2} \times [(A-3)W + \sum_i \phi_i \frac{\partial W}{\partial \phi_i} + \text{h.c.}]$ , where  $A$  is a dimensionless number associated with hidden sector dynamics [24]. Since dimensions of the operators associated with the  $\lambda_k$ ’s ( $k = 1, 2, 3, 4$ ) in Eq. (3) are all the same, the “A-term” coefficients ( $\equiv A_{\lambda_k}$ ) corresponding to  $\lambda_k$  should be  $m_{3/2}(A+1)\lambda_k$ , and so satisfy  $A_{\lambda_{j+1}}/A_{\lambda_j} = \lambda_{j+1}/\lambda_j$  ( $j = 1, 3$ ). Hence, at the minimum, the “A-terms” corresponding to  $\lambda_k$  are cancelled by each other with the VEVs in Eq. (5). Since the other soft terms are symmetric under  $P \leftrightarrow \overline{P}$  and  $Q \leftrightarrow \overline{Q}$ , we have  $\langle P \rangle = \langle \overline{P} \rangle$  and  $\langle Q \rangle = \langle \overline{Q} \rangle$  at the minimum of the scalar potential.

<sup>7</sup>In gauge mediated SUSY breaking scenario, “A-terms” are generally suppressed.

Since the fields that couple to  $P$ ,  $\overline{P}$ ,  $Q$  and  $\overline{Q}$  are all superheavy, the soft parameters are expected to be radiatively stable at low energies. Thus, at the minimum of the scalar potential, we have four mass eigen states,  $(P \pm \overline{P})/\sqrt{2}$  ( $\equiv P_{\pm}$ ) and  $(Q \pm \overline{Q})/\sqrt{2}$  ( $\equiv Q_{\pm}$ ). While  $P_+$  and  $Q_+$  obtain superheavy masses of order  $M_{GUT}$  and large VEVs ( $= \sqrt{\kappa_{1,2}/\rho_{1,2}} M_* + O(m_{3/2}) \sim M_{GUT}$ , respectively),  $P_-$  and  $Q_-$  remain light ( $\sim m_{3/2}$ ) with vanishing VEVs.

With  $\langle \mathbf{45}_H \rangle$  in Eq. (2), the ‘‘DT splitting problem’’ resolves itself through the mechanism in [17]. Consider the superpotential:

$$W_{10} = y_1 \mathbf{1045}'_H \mathbf{10} + y_2 \mathbf{1045}_H \mathbf{10}_h . \quad (10)$$

From the first term in Eq. (10), only the doublets contained in  $\mathbf{10}$  become superheavy [21], and from the second term only the color triplet fields included in  $\mathbf{10}$  and  $\mathbf{10}_h$  acquire superheavy masses [17, 20, 21]. Since the two color triplets contained in  $\mathbf{10}_h$  do not couple in Eq. (10), dimension five nucleon decay which may be in conflict with the Superkamiokande observations [15] is eliminated in the SUSY limit [21]. Note that operators such as  $\mathbf{1010}_h$ ,  $\mathbf{10}_h \mathbf{10}_h$ ,  $[\mathbf{1010}_h] \text{Tr}(\mathbf{45}_H \mathbf{45}_H)$  and so on are allowed by  $SO(10)$  and, unless forbidden, would destroy the gauge hierarchy. The  $U(1)$  symmetries in Table I are once again crucial in achieving this.

Although the superpotential coupling  $\langle S \rangle \mathbf{10}_h \mathbf{10}_h$  induces higgsino mediated dimension five nucleon decay, there is a huge suppression factor of  $m_{3/2}/M_{GUT}$ . Thus, we expect that nucleon decay is dominated by the exchange of the superheavy gauge bosons with an estimated lifetime  $\tau_p \rightarrow e^+ \pi^0$  of order  $10^{34} - 10^{36}$  yrs. Note that we have assumed that dimension five operators such as  $\mathbf{16}_i \mathbf{16}_j \mathbf{16}_k \mathbf{16}_l$ ,  $\mathbf{16}_i \mathbf{16}_j \mathbf{16}_k \mathbf{16}_H$  and so on, where the subscripts are family indices of the matter, are adequately suppressed by assigning suitable  $R$  and  $A$  charges to these matter superfields. This is closely tied to the flavor problem, which we will not address here.

Consider next the superpotential couplings involving the third generation matter



superfields,

$$W_m = y_3 \mathbf{16}_3 \mathbf{16}_3 \mathbf{10}_h + \frac{y_\nu}{M_*} \mathbf{16}_3 \mathbf{16}_3 \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H . \quad (11)$$

The first term yields Yukawa unification so that the MSSM parameter  $\tan\beta \approx m_t/m_b$  [27]. For a realistic construction of the fermion's mass matrices in  $SO(10)$ , refer to e.g. Ref. [28]. From the  $y_\nu$  term, the right handed neutrino masses are  $\lesssim y_\nu M_{B-L}^2/M_* \sim 10^{14}$  GeV. Right handed neutrino masses of order  $10^{14}$  GeV and smaller can yield a mass spectrum for the light neutrinos through the seesaw mechanism, that is suitable for neutrino oscillations. These masses are also appropriate for realizing leptogenesis after inflation [29, 7]. Finally let us note that the  $\mathbf{16}_H, \overline{\mathbf{16}}_H$  VEVs break the center  $Z_4$  of  $SO(10)$  completely [22]. The role of ‘matter parity’ is played by the unbroken  $Z_2$  subgroup of the  $U(1)$   $R$ -symmetry [9]. Thus the LSP in our model is expected to be stable and contribute to the dark matter in the universe.

For completeness, we need to present also the other possible terms in the superpotential that were not discussed in Eqs. (1), (3), (8), (10), and (11). Indeed, we have more quartic couplings;  $\mathbf{1045} \mathbf{10}_h X$ ,  $\mathbf{16} \overline{\mathbf{16}} X Q$ ,  $\mathbf{16} \mathbf{16}_H \mathbf{10} X$ ,  $\mathbf{16}' \mathbf{16}_H \mathbf{10}_h X$ ,  $\mathbf{16}_H \mathbf{16}_H \mathbf{10}_h S$ ,  $\mathbf{16}_H \mathbf{16}_H \mathbf{1045}_H$ ,  $\overline{\mathbf{16}}' \overline{\mathbf{16}}_H \mathbf{10}_h X$ ,  $\overline{\mathbf{16}}_H \overline{\mathbf{16}}_H \mathbf{10}_h S$ ,  $\overline{\mathbf{16}}_H \overline{\mathbf{16}}_H \mathbf{1045}_H$ , and so on, which also are consistent with the charge assignment in Table I. But they just provide sub-dominant effects in this model. For instance,  $\mathbf{16} \overline{\mathbf{16}} \langle X Q \rangle$  can not change the symmetry breaking pattern discussed above, because  $\langle \mathbf{16} \rangle = \langle \overline{\mathbf{16}} \rangle = 0$ . Thus, with keeping massless goldstones, it just modifies the masses of the pseudo-goldstone modes contained in  $\mathbf{16}$  and  $\overline{\mathbf{16}}$ . Due to  $\langle \mathbf{16}_H \rangle \neq 0$  and  $\langle \mathbf{45}_H \rangle \neq 0$ ,  $\mathbf{16}_H \mathbf{16}_H \mathbf{1045}_H$  slightly changes masses of the  $SU(2)_L$  doublets contained in  $\mathbf{16}_H$  and  $\mathbf{10}$ . We also have over forty extra penta-couplings except those considered in Eqs. (3) and (8), but we will neglect them.

Let us now discuss how inflation is implemented in the model described so far. In particular, we aim to show that the SUGRA corrections do not significantly affect the inflationary scenario, which is a non-trivial result in inflationary model building.

The “F-term” scalar potential in SUGRA is given by

$$V_F = e^{K/M_P^2} \left[ \sum_{i,j} (K^{-1})_j^i (D_{\phi_i} W) (D_{\phi_j} W)^* - 3 \frac{|W|^2}{M_P^2} \right], \quad (12)$$

where  $M_P (\equiv M_{\text{Planck}}/\sqrt{8\pi} = 2.4 \times 10^{18} \text{ GeV})$  denotes the reduced Planck mass.  $K (= K(\phi_i, \phi_j^*) = K^*)$  and  $W (= W(\phi_i))$  are the Kähler potential and the superpotential, respectively.  $(K^{-1})_j^i$  in Eq. (12) denotes the inverse of  $\partial^2 K / \partial \phi_i \partial \phi_j^*$ . In our case,  $W$  is composed of Eqs. (1), (3), (8), (10), and (11).  $D_{\phi_i} W$  in Eq. (12) is defined as

$$D_{\phi_i} W \equiv \frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} \frac{W}{M_P^2}. \quad (13)$$

The Kähler potential can be expanded as  $K = |\phi_i|^2 + c_4 |\phi_i|^4 / M_P^2 + \dots$ . For simplicity, we consider the minimal case with  $\partial^2 K / \partial \phi_i \partial \phi_j^* = \delta_j^i$ . Indeed, as explained in [2], higher order terms in  $K$  (with a coefficient  $\lesssim 10^{-2}$  for the quartic term) do not seriously affect inflation. For simplicity, we will also ignore the TeV scale electroweak symmetry breaking effects when discussing inflation.

In this paper, we aim to employ the ‘shifted’ hybrid inflationary scenario proposed in Ref [10], in which symmetries can be broken during inflation unlike the simple “hybrid inflation” model [1]. An inflationary scenario is realized in the early universe with the scalar fields  $S$ ,  $\mathbf{16}_H$ ,  $\overline{\mathbf{16}}_H$ ,  $P$ ,  $\overline{P}$ ,  $Q$ , and  $\overline{Q}$  displaced from the present values. We suppose that initially  $|\langle S \rangle|^2 \gtrsim M_{B-L}^2 [1/(4\zeta) - 1]/2$  with  $1/4 < \zeta < 1/7.2$  [10], and  $\langle \mathbf{16}_H \rangle, \langle \overline{\mathbf{16}}_H \rangle, \langle P \rangle, \langle \overline{P} \rangle, \langle Q \rangle, \langle \overline{Q} \rangle \neq 0$  with the inflationary superpotential given by [10],

$$\begin{aligned} W_{\text{infl}} &\approx -\kappa S \left[ M_{B-L}^2 - \mathbf{16}_H \overline{\mathbf{16}}_H + \frac{\rho}{\kappa M_*^2} (\mathbf{16}_H \overline{\mathbf{16}}_H)^2 \right. \\ &\quad \left. - \frac{\kappa_1}{\kappa} P \overline{P} + \frac{\rho_1}{\kappa M_*^2} (P \overline{P})^2 - \frac{\kappa_2}{\kappa} Q \overline{Q} + \frac{\rho_2}{\kappa M_*^2} (Q \overline{Q})^2 \right] \\ &\equiv -\kappa S M_{\text{eff}}^2, \end{aligned} \quad (14)$$

where  $M_{\text{eff}}^2$  turns out to be of order  $M_{B-L}^2$ . With  $D_S W \approx -\kappa M_{\text{eff}}^2 (1 + |S|^2/M_P^2)$ , one can see that the “F-term” scalar potential becomes

$$V_F \approx \left( 1 + \sum_k \frac{|\phi_k|^2}{M_P^2} + \dots \right) \left[ \kappa^2 M_{\text{eff}}^4 \left( 1 + \frac{|S|^4}{2M_P^4} \right) + \left( 1 + \frac{|S|^2}{M_P^2} + \frac{|S|^4}{2M_P^4} \right) \sum_k |D_{\phi_k} W|^2 \right], \quad (15)$$

where all scalar fields except  $S$  contribute to  $\phi_k$ . The factor  $(1 + \sum_k |\phi_k|^2/M_P^2 + \dots)$  in front originates from  $e^{K/M_P^2}$  in Eq. (12). In Eq. (15) the quadratic term in  $S$  from  $|D_S W|^2$ , which is of order  $(\kappa^2 M_{\text{eff}}^4/M_P^2)|S|^2$  ( $\approx H^2|S|^2$ ), has canceled out with the factor “ $-3|W|^2/M_P^2$ ” ( $\approx -3\kappa^2 M_{\text{eff}}^4|S|^2/M_P^2$ ) and the quadratic term in  $S$  from “ $e^{K/M_P^2}$ ” ( $= 1 + |S|^2/M_P^2 + \dots$ ). It is a common feature in this class of models [2]. Thus, only if  $|D_{\phi_k} W|/M_P$ 's are much smaller than the Hubble scale ( $\sim \kappa M_{\text{eff}}^2/M_P$ ), the flatness of  $S$  will be guaranteed even with the SUGRA corrections included. Note that the  $U(1)$   $R$ -symmetry ensures the absence of terms proportional to  $S^2$ ,  $S^3$ , etc. in the superpotential, which otherwise could spoil the slow-roll conditions.

Let us consider the inflationary trajectory on which  $\langle \mathbf{10} \rangle = \langle \mathbf{10}_h \rangle = \langle \mathbf{16} \rangle = \langle \overline{\mathbf{16}} \rangle = \langle \mathbf{16}' \rangle = \langle \overline{\mathbf{16}}' \rangle = \langle \mathbf{16}_3 \rangle = \langle \mathbf{45} \rangle = 0$ , with  $D_{\mathbf{10}} W = D_{\mathbf{10}_h} W = D_{\mathbf{16}} W = D_{\overline{\mathbf{16}}} W = D_{\mathbf{16}'} W = D_{\overline{\mathbf{16}}'} W = D_{\mathbf{16}_3} W = D_{\mathbf{45}} W = 0$ . On the other hand,

$$D_{\mathbf{16}_H} W = \kappa S \left[ \overline{\mathbf{16}}_H \left( 1 - \frac{2\rho}{\kappa M_*^2} (\mathbf{16}_H \overline{\mathbf{16}}_H)^2 \right) - \mathbf{16}_H^* \frac{M_{\text{eff}}^2}{M_P^2} \right], \quad (16)$$

$$D_P W = \kappa S \left[ \overline{P} \left( \frac{\kappa_1}{\kappa} - \frac{2\rho_1}{\kappa M_*^2} (P \overline{P})^2 \right) - P^* \frac{M_{\text{eff}}^2}{M_P^2} \right], \quad (17)$$

$$D_Q W = \kappa S \left[ \overline{Q} \left( \frac{\kappa_2}{\kappa} - \frac{2\rho_2}{\kappa M_*^2} (Q \overline{Q})^2 \right) - Q^* \frac{M_{\text{eff}}^2}{M_P^2} \right], \quad (18)$$

and similarly  $D_{\overline{\mathbf{16}}_H} W = D_{\mathbf{16}_H} W (\mathbf{16}_H \leftrightarrow \overline{\mathbf{16}}_H)$ ,  $D_{\overline{P}} W = D_P W (P \leftrightarrow \overline{P})$ , and  $D_{\overline{Q}} W = D_Q W (Q \leftrightarrow \overline{Q})$ . The other  $D_{\phi_l} W$ 's ( $\phi_l = X^{(\prime)}$ ,  $Y$ ,  $\mathbf{45}_H$ ,  $\mathbf{45}'_H$ ) are approximately given by  $-s \langle \phi_l^* \rangle$ , where  $s \equiv -W/M_P^2 \approx \kappa \langle S \rangle M_{\text{eff}}^2/M_P^2$  ( $\ll M_{GUT}$ ). At one of the local minima,  $\langle \mathbf{16}_H \rangle$ ,  $\langle \overline{\mathbf{16}}_H \rangle$ ,  $\langle P \rangle$ ,  $\langle \overline{P} \rangle$ ,  $\langle Q \rangle$ ,  $\langle \overline{Q} \rangle$ , and the vacuum energy  $V_0^{1/4}$  acquire the following values,

$$|\langle \mathbf{16}_H \rangle|^2 = |\langle \overline{\mathbf{16}}_H \rangle|^2 \approx \frac{\kappa M_*^2}{2\rho} \left[ 1 - \frac{M_{B-L}^2}{M_P^2} + \frac{\kappa M_*^2}{2\rho M_P^2} \left( 1 - \frac{M_{B-L}^2}{4S^2} + O(\kappa_{1,2}^2/\kappa^2) \right) \right], \quad (19)$$

$$|\langle P \rangle|^2 = |\langle \overline{P} \rangle|^2 \approx \frac{\kappa_1 M_*^2}{2\rho_1} \left[ 1 - \frac{\kappa M_{B-L}^2}{\kappa_1 M_P^2} + \frac{\kappa^2 M_*^2}{4\kappa_1 \rho M_P^2} \left( 1 + O(\kappa_{1,2}^2/\kappa^2) \right) \right], \quad (20)$$

$$|\langle Q \rangle|^2 = |\langle \overline{Q} \rangle|^2 \approx \frac{\kappa_2 M_*^2}{2\rho_2} \left[ 1 - \frac{\kappa M_{B-L}^2}{\kappa_2 M_P^2} + \frac{\kappa^2 M_*^2}{4\kappa_2 \rho M_P^2} \left( 1 + O(\kappa_{1,2}^2/\kappa^2) \right) \right], \quad (21)$$

$$V_0 \approx \kappa^2 M_0^4 \left[ 1 + \frac{M_*^2}{M_P^2} \left( \frac{\kappa}{\rho} + \frac{\kappa_1}{\rho_1} + \frac{\kappa_2}{\rho_2} + O(\kappa^2, \kappa M_{B-L}^2/M_P^2) \right) + \sum_l \frac{|\langle \phi_l \rangle|^2}{M_P^2} \right], \quad (22)$$

where we assumed  $\frac{\kappa}{\rho} \gtrsim \frac{\kappa_1}{\rho_1}, \frac{\kappa_2}{\rho_2}$  with  $\kappa \ll 1$  and  $\rho \sim \rho_1 \sim \rho_2 \sim O(1)$ . In Eq. (22),  $M_0^4 \equiv M_{B-L}^4 [1/(4\zeta) + 1/(4\zeta_1) + 1/(4\zeta_2) - 1]^2$  ( $\approx M_{B-L}^4 [1/(4\zeta) - 1]^2$ ), where  $\zeta_1 \equiv \rho_1 M_{B-L}^2 / (\kappa_1 M_*^2)$ , and  $\zeta_2 \equiv \rho_2 M_{B-L}^2 / (\kappa_2 M_*^2)$ . Eqs. (19)–(22) are valid only when  $M_{B-L}^2 |1/(4\zeta) - 1|/2 \lesssim |\langle S \rangle|^2 \ll M_P^2$ .<sup>8</sup> In the limit  $M_P \rightarrow \infty$ , the above results approach the values in global SUSY [10].

Since  $P$  and  $Q$  develop VEVs,  $X^{(\prime)}, Y, \mathbf{45}_H$ , and  $\mathbf{45}'_H$  should also achieve VEVs from  $D_{\mathbf{16}^{(\prime)}} W = D_{\overline{\mathbf{16}}^{(\prime)}} W = 0$  even during inflation. Consequently,  $SO(10)$  and  $U(1)_A$  are broken to  $G_{SM}$  during inflation. Note that  $\langle P \rangle = \langle \overline{P} \rangle$  and  $\langle Q \rangle = \langle \overline{Q} \rangle$  in Eqs. (20) and (21) lead to  $\langle P_- \rangle = \langle Q_- \rangle = 0$ . A non-zero vacuum energy from the “F-term” potential induces universal “Hubble induced scalar mass terms” ( $\kappa^2 M_0^4 / M_P^2 \times |\phi_l|^2$ ), which are read off from Eq. (15). But such small masses ( $\kappa M_0^2 / M_P \ll M_{B-L}$ ) cannot much affect the VEVs of the superheavy scalars of order  $M_{GUT}$ .

Indeed, as seen earlier, in the SUSY limit the VEVs of  $P, \overline{P}, Q, \overline{Q}$  are not determined, even though  $\langle P\overline{P} \rangle$  and  $\langle Q\overline{Q} \rangle$  are fixed. But by including the SUSY breaking soft terms of order  $m_{3/2}$  in the scalar potential, they are completely determined. Thus, one might expect that the non-vanishing VEV of  $S$  and the “Hubble induced masses” ( $\gg m_{3/2}$ ) during inflation cause the VEVs of  $P, \overline{P}, Q, \overline{Q}$  to significantly deviate from their values at low energies. Such differences, if true, would result in oscillations by  $P, \overline{P}, Q$ , and  $\overline{Q}$  (or  $P_{\pm}$  and  $Q_{\pm}$ ) after inflation. As explained earlier, with universal soft masses,  $\langle P_- \rangle = \langle Q_- \rangle = 0$ . Since the VEVs of  $P_-$  and  $Q_-$  vanish both during and after inflation, oscillations by such light ( $\sim m_{3/2}$ ) scalars would not arise after inflation has ended.

A mass term for  $S$  is induced by SUGRA corrections, such that the “F-term potential” contains

$$V_F \supset \sum_l |D_{\phi_l} W|^2 \sim \left( \frac{M_{GUT}}{M_P} \right)^2 \times H^2 |S|^2, \quad (23)$$

where  $H$  ( $\equiv \kappa M_{\text{eff}}^2 / M_P \approx \kappa M_0^2 / M_P$ ) denotes the “Hubble induced mass.” Such a

<sup>8</sup>  $\mathbf{16}_H$  and  $\overline{\mathbf{16}}_H$  develop the VEVs in the neutrino directions  $\langle \nu_H^c \rangle, \langle \bar{\nu}_H^c \rangle$ . Near the VEVs during inflation, the normalized real scalar fields,  $\text{Re}(\delta \nu_H^c + \delta \bar{\nu}_H^c)$  and  $\text{Im}(\delta \nu_H^c - \delta \bar{\nu}_H^c)$  acquire mass squareds given by  $m_{\pm}^2 \approx 4\kappa^2 |\langle S \rangle|^2 \pm 2\kappa^2 M_{B-L}^2 [1/(4\zeta) - 1]$ , respectively [10].

small mass term of  $S$  ( $\ll H^2|S|^2$ ) does not spoil the slow roll conditions. The correction term in Eq. (23) has a small impact on the inflationary predictions.

With SUSY broken during inflation ( $F_S \neq 0$ ), there are radiative corrections from the  $\mathbf{16}_H$ ,  $\overline{\mathbf{16}}_H$  supermultiplets, which provide logarithmic corrections to the tree level potential  $V_F \approx \kappa^2 M_0^4$ , and thereby drive inflation [1]. In our model, the scalar spectral index turns out to be  $n_s = 0.99 \pm 0.01$  for  $\kappa < 10^{-2}$ . (See FIG. 1.) The symmetry breaking scale  $M_{B-L}$  is estimated to be around  $10^{16} - 10^{17}$  GeV (FIG. 2).

Before concluding, some remarks about the reheat temperature  $T_r$ , leptogenesis, and right handed neutrino masses are in order. When inflation is over, the inflatons decay into handed neutrinos. Following Refs. [4] and [30], a lower bound on  $T_r$  is obtained for  $\kappa = \lambda$ , and the results are summarized in FIG.3. (To obtain FIG.1-3, we set  $M_* = M_P$  and  $\rho = \rho_{1,2} = 1 \gg \frac{\kappa_{1,2}^2}{\kappa^2}$ .) We see that  $T_r \lesssim 10^9$  GeV for  $\kappa \lesssim 10^{-2}$ . The inflaton decay into right handed neutrinos yields the observed baryon asymmetry via leptogenesis. Assuming non-thermal leptogenesis and hierarchical right handed neutrinos, we estimate the three right handed neutrinos masses to be of order  $10^{14}$  GeV,  $(10 - 20) \times T_r$  and  $\text{few} \times T_r$ . Note that with  $\kappa < 10^{-2}$  the inflaton (with mass  $\sim \sqrt{\kappa M_{B-L}^2}$ ) can not decay into the heaviest right handed neutrino (of mass  $\sim 10^{14}$  GeV). Thus, the latter does not play a direct role in leptogenesis.

In summary, our goal here was to realize inflation in a realistic SUSY  $SO(10)$  model. A global  $U(1)_A$  and the  $U(1)$   $R$ -symmetry play essential roles in the analysis. Several testable predictions emerge. In particular, the scalar spectral index  $n_s = 0.99 \pm 0.01$ , which will be tested by several ongoing experiments. Proton decay proceeds via  $e^+\pi^0$ , with an estimated lifetime of order  $10^{34} - 10^{36}$  yrs. The LSP is stable and the MSSM parameter  $\tan\beta$  is large, of order  $m_t/m_b$ . Two of the three right handed neutrino masses are fairly well determined. The heaviest one weighs around  $10^{14}$  GeV, and the one primarily responsible for non-thermal leptogenesis has mass of order  $10 T_r$ , where the reheat temperature  $T_r$  is around  $10^8 - 10^9$  GeV.

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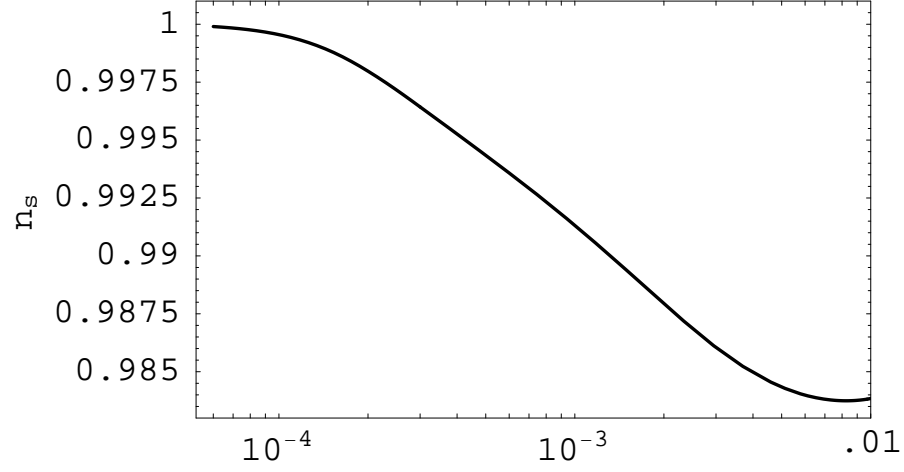


FIG. 1: The spectral index  $n_s$  vs  $\kappa$ .  $\kappa$  is  $< 0.01$  so that the reheat temperature does not exceed  $10^9$  GeV. See FIG. 3.

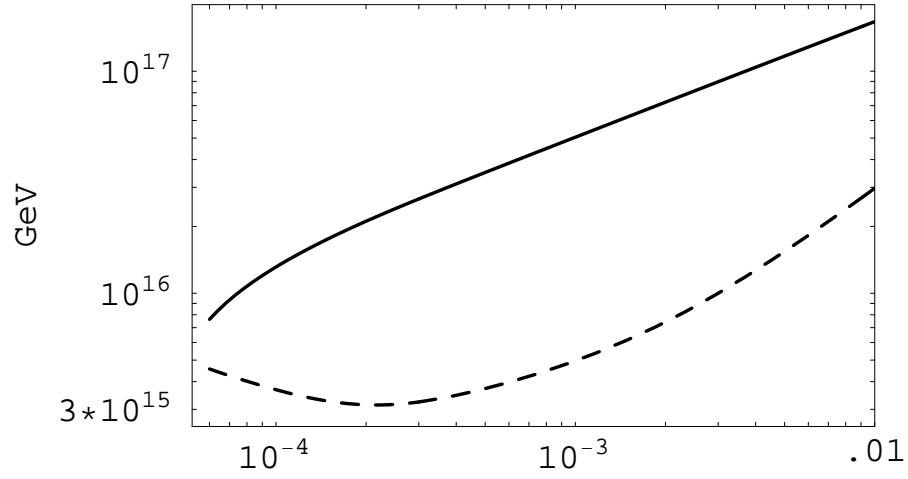


FIG. 2: The symmetry breaking scale  $M_{B-L}$  (solid) and magnitude of the inflaton  $|S|$  (dashed) vs  $\kappa$ .

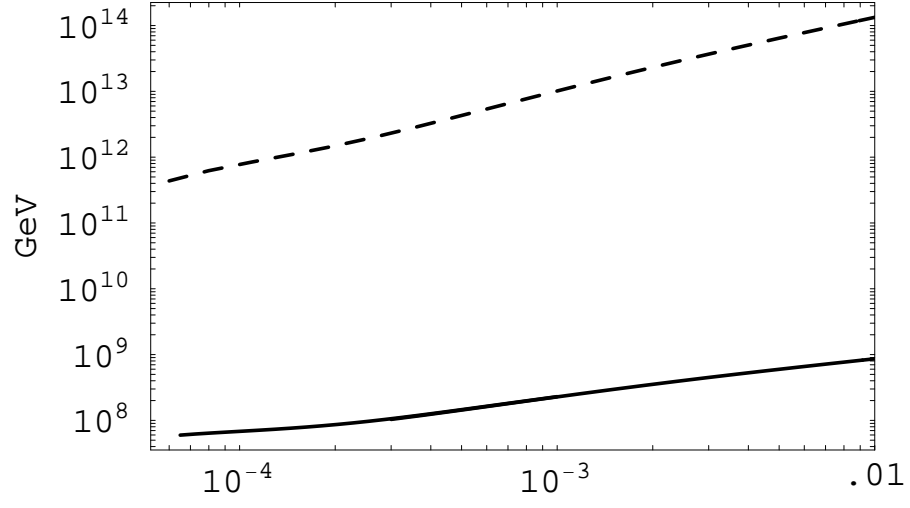


FIG. 3: Reheat temperature  $T_r$  and inflaton mass (dashed) vs  $\kappa$ .